FIU
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Information Theoretic Inequality System Prover

Students: Alfredo Cabanas
Mentor: *Dr. Farhad Shirani Chaharsooghi, PhD, Florida International University, KFSCIS*
Instructor: *Dr. Masoud Sadjadi, PhD*, Florida International University

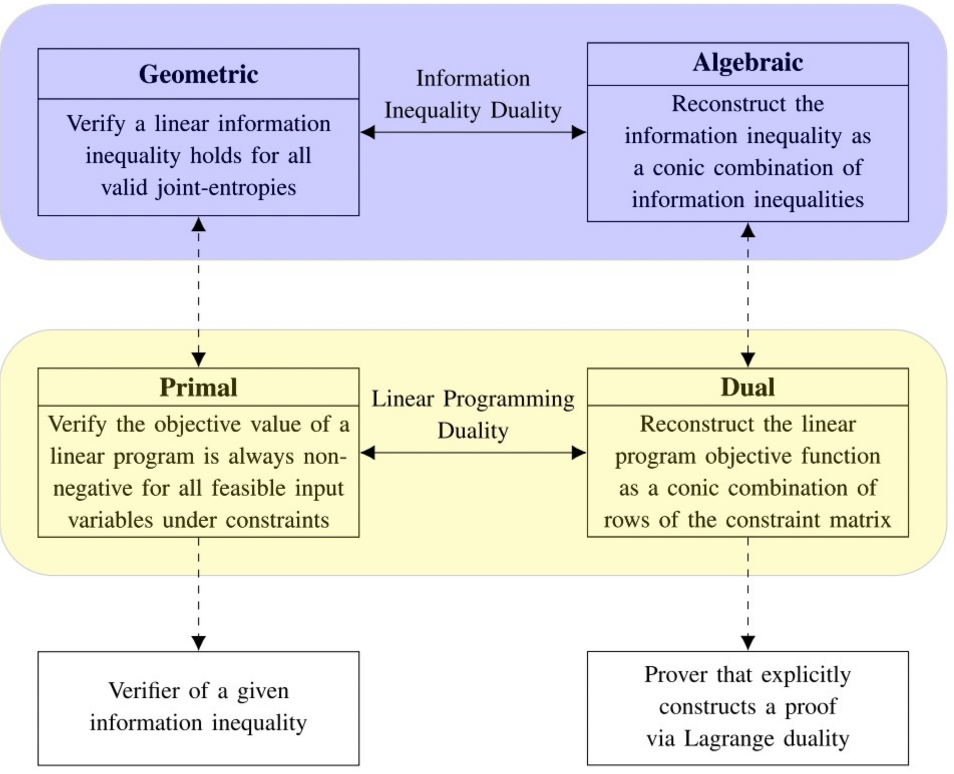
Motivation

- Quantifying information important in communications, cybersecurity, and machine learning, etc.
- Information measures: entropy, conditional entropy, and mutual information.
- **Objective:** Evaluate consistency of given system of information inequalities.
- **Example:** We wish to check the consistency of

$$H(X_1) + H(X_2) \geq \frac{3}{2} H(X_1, X_2)$$
$$H(X_1) + H(X_3) \geq \frac{3}{2} H(X_1, X_3)$$
$$H(X_1, X_2) + H(X_1, X_3) \geq H(X_1) + 2H(X_2) + 2H(X_3)$$

Prior Works

1. ITIP- **Language:** Matlab, online prover available
   Based on: Yeung, R.W. (1997). "A framework for linear information inequalities"
   Feature: First proof checker of this kind.
2. XITIP- **Language:** C, **Based on:** improvements made by a group at EPFL, **Feature:** Faster than ITIP
3. oXITIP- **Language:** GLPK optimizer with C++ backend, **Feature:** web-based user interface.
4. AITIP- **Language:** Gurobi solver for optimization.
   Application Domain: Single Inequalities, **Feature:** Web-based service. Provides canonical break down of the inequalities in terms of basic Info measures.



All prior works investigate single inequalities.

Fourier-Motzkin Elimination

- Fourier Motzkin Elimination maps linear systems of inequalities to lower dimensions.
- ***Interpretation:*** Projecting polytopes on hyperplanes.

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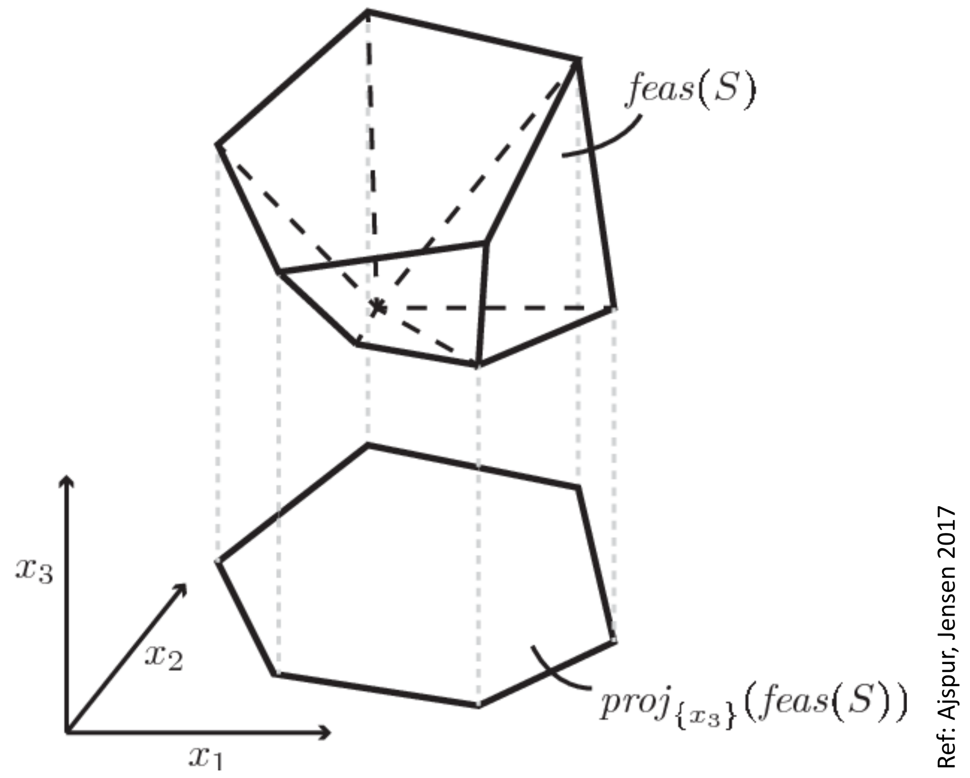

Input: System of inequalities $A^{m \times n} x^n \leq b^m$
1. Choose one of the variables to eliminate.
2. Partition the inequalities into those with positive and negative coefficients for this variable.
3. For each pair of inequalities, multiply both sides by a positive coefficient and combine to eliminate the desired variable.
4. Repeat steps 2-4 until all variables have been eliminated.
5. The system is consistent if the result is not a contradiction.

Shannon-Type Inequalities

The following Inequalities are always true, and are called Shannon-type inequalities:

$$I(X_A; X_B | X_C) \geq 0, \forall A, B, C \subseteq [m]$$
$$\Rightarrow H(X_A, X_B) + H(X_B, X_C) - H(X_C)$$
$$- H(X_A X_B X_C) \geq 0, , \forall A, B, C \subseteq [m]$$

- Any system of inequalities should be consistent with the above.
- Any system of inequalities which is produced by positive linear combinations of the above is true.

Deliverables and Outcomes

Main Idea: Use Fourier-Motzkin Elimination to verify whether a given system of inequalities is inconsistent with Shannon-type inequalities, or whether it is a positive linear combination of these inequalities.

Step 1: Generating all Shannon-type inequalities:

```
H([][2]) + H([2][2, 3]) - H([][2][2, 3]) - H([2, 3]) >= 0
H([][2]) + H([2][1, 2, 3]) - H([][2][1, 2, 3]) - H([1, 2, 3]) >= 0
H([][1, 2]) + H([1, 2][1]) - H([][1, 2][1]) - H([1]) >= 0
H([][1, 2]) + H([1, 2][2]) - H([][1, 2][2]) - H([2]) >= 0
H([][1, 2]) + H([1, 2][3]) - H([][1, 2][3]) - H([3]) >= 0
H([][1, 2]) + H([1, 2][1, 3]) - H([][1, 2][1, 3]) - H([1, 3]) >= 0
H([][1, 2]) + H([1, 2][2, 3]) - H([][1, 2][2, 3]) - H([2, 3]) >= 0
H([][1, 2]) + H([1, 2][1, 2, 3]) - H([][1, 2][1, 2, 3]) - H([1, 2, 3]) >= 0
H([][3]) + H([3][1]) - H([][3][1]) - H([1]) >= 0
H([][3]) + H([3][2]) - H([][3][2]) - H([2]) >= 0
H([][3]) + H([3][1, 2]) - H([][3][1, 2]) - H([1, 2]) >= 0
H([][3]) + H([3][1, 3]) - H([][3][1, 3]) - H([1, 3]) >= 0
H([][3]) + H([3][2, 3]) - H([][3][2, 3]) - H([2, 3]) >= 0
H([][3]) + H([3][1, 2, 3]) - H([][3][1, 2, 3]) - H([1, 2, 3]) >= 0
```

Step 2: Append new system and run Fourier-Motzkin:

- FM algorithm removing any solved or unviable inequalities of the generated list

```
PROBLEMS    OUTPUT    DEBUG CONSOLE    TERMINAL

solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
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solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
solved a linear program and removed a row
```

```
A_new= [[nan]
 [ 1.]]
b_new= [[-inf]
 [  0.]]
The inequality is incorrect
```

- Final results appear as input array is returned as [nan]

REFERENCES

Research materials were provided by Dr. Farhad Shirani Chaharsooghi of Florida International University.

Cover, T. M., & Thomas, J. A. (1991). *Elements of Information Theory*. John Wiley & Sons, INC.

F. S. Chaharsooghi, M. J. Emadi, M. Zamanighomi and M. R. Aref, "A new method for variable elimination in systems of inequations," 2011 IEEE International Symposium on Information Theory Proceedings, 2011, pp. 1215-1219, doi: 10.1109/ISIT.2011.6033728.

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